K25P 1901

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.C.S.S.-OBE – Reg./Supple./Imp.) Examination, April 2025
(2023 and 2024 Admissions)
MATHEMATICS
MSMAT02C09/MSMAF02C09 : Advanced Topology

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **any five** questions. **Each** question carries **4** marks.

1. Define compact space. Prove that the real line $\mathbb{R}$ is not compact.

2. Prove that every closed interval in $\mathbb{R}$ is uncountable.

3. Prove that a subspace of a first countable space is first countable.

4. Prove that the space $\mathbb{R}_l$ is normal.

5. Prove or disprove : A completely regular space is regular.

6. Let $A \subset X$; let $f : A \to \mathbb{Z}$ be a continuous map of A into the Hausdorff space $\mathbb{Z}$. Then prove that there is at most one extension of f to a continuous function $g : \bar{A} \to \mathbb{Z}$. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Prove that every nonempty finite ordered set has the order type of a section {1, ...,n} of $\mathbb{Z}_+$, so it is well-ordered.

8. Let X be a metrizable space. If X is limit point compact prove that X is sequentially compact.

9. Prove that the space $\mathbb{R}_l$ satisfies all the countability axioms but the second.

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10. Prove that the Sorgenfrey plane $\mathbb{R}_l^2$ is not normal.

11. Let X be a completely regular space. If Y_1 and Y_2 are two compactifications of X satisfying the extension property, then prove that Y_1 and Y_2 are equivalent. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. a) Let X be a Hausdorff space. Then prove that X is locally compact if and only if given x in X and given a neighborhood U of x, there is a neighborhood V of x such that $\bar{V}$ is compact and $\bar{V} \subset U$.

    b) State and prove the Uniform Continuity Theorem.

13. a) Prove that every regular space with a countable basis is normal.

    b) Prove that every metrizable space is normal.

14. State and prove Urysohn lemma.

15. State and prove Tietze Extension Theorem.

16. Let X be a set ; let $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Then prove the following :

    a) Any finite intersection of elements of $\mathcal{D}$ is an element of $\mathcal{D}$.

    b) If A is a subset of X that intersects every element of $\mathcal{D}$, then A is an element of $\mathcal{D}$. **(3×13=39)**

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